Class $\mathbf{- 1 2}^{\text {th }}$
Worksheet-14

Chapter-4

1. Prove that $\left|\begin{array}{lll}1 & x & x^{2} \\ 1 & y & y^{2} \\ 1 & z & z^{2}\end{array}\right|=(x-y)(y-z)(z-x)$.
2. Without expanding, prove that

$$
\Delta=\left|\begin{array}{ccc}
b+c & c+a & a+b \\
q+r & r+p & p+q \\
y+z & z+x & x+y
\end{array}\right|=2\left|\begin{array}{ccc}
a & b & c \\
p & q & r \\
x & y & z
\end{array}\right| .
$$

3. Evaluate the determinant $\left|\begin{array}{lll}1 / a & a^{2} & b c \\ 1 / b & b^{2} & c a \\ 1 / c & c^{2} & a b\end{array}\right|$
4. Prove that $\left|\begin{array}{ccc}a+b+2 c & a & b \\ c & b+c+2 a & b \\ c & a & c+a+2 b\end{array}\right|=2(a+b+c)^{3}$
5. Prove that $\left|\begin{array}{ccc}1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c\end{array}\right|=a b c\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$.
6. Solve the equation $\left|\begin{array}{ccc}x+a & b & c \\ c & x+b & a \\ a & b & x+c\end{array}\right|=0$
7. Prove that

$$
\left|\begin{array}{ccc}
x & y & z \\
x^{2} & y^{2} & z^{2} \\
y z & z x & x y
\end{array}\right|=\left|\begin{array}{ccc}
1 & 1 & 1 \\
x^{2} & y^{2} & z^{2} \\
x^{3} & y^{3} & z^{3}
\end{array}\right|=(y-z)(z-x)(x-y)(y z+z x+x y) .
$$

