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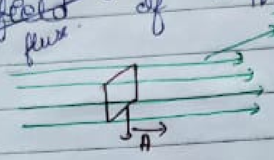
Chapter - 2
Gauss's law and its application.

Electric flux → when number of electric field lines passing through a given surface is known as ~~field~~ flux of that surface

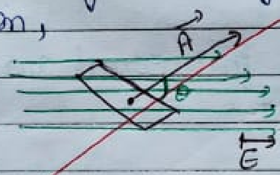


It is represented by "ϕ". If electric field lines are passing through the unit surface and area of given surface A.

then. $\phi = EA$

Here electric flux is equal to product of electric field and its area.

* If surface is making an angle θ with electric field lines then,



Electric flux is equal to electric field and effective area by which field lines are passing.

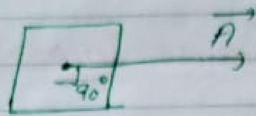
hence $\boxed{\phi = E \cdot A\cos\theta}$

where $A\cos\theta$ is effective area and θ angle b/w $\vec{A}$ and $\vec{E}$

$$\boxed{\phi = \vec{E}\cdot\vec{A}}$$

thus, we can say that electric flux is dot product of area vector and electric field vector.

Area vector → As we known that area is a scalar quantity but in physics we treat area as a vector quantity where magnitude is equal and direction is perpendicular outward to the surface.



Area = magnitude

Unit → $\phi = EA$

$\phi = \frac{N}{C} m^2$ $\quad [\because E = \frac{F}{q}]$

$\phi = \frac{N \times m \cdot m}{C}$ $\quad [\because N \times m = F \times S = W]$

$\phi = \frac{Joule}{C} \cdot m$ $\quad [\frac{J}{C} = Volt]$

$\phi = Volt\ m.$

Dimension → $\phi = EA$

$\phi = \frac{F}{q} \cdot m$

$\phi = \frac{N}{C} \cdot m$

$\phi = \frac{M^1 L^1 T^{-2}}{A^1 T^1} \cdot L^2$

$\phi = M^1 L^3 T^{-3} A^{-1}$

Note → If angle θ is an ~~so~~ angle substanded by surface and Electric field then angle substanded by $\vec{A}$ and E. field will be →

$(90-\theta)$.

$\phi = E \cdot A \cos(90-\theta)$ (θ is angle b/w Surface and E.F)

$\phi = E \cdot A \sin\theta$

Gauss law

→ A/c to gauss law electric flux associated with a Surface which includes some quantity of charge is in ratio of sum of charges to the ε_0 (Electric permeablitity)

$$\boxed{\phi = \frac{\Sigma Q}{\varepsilon_0}}$$

Properties

① ΣQ is the algebric sum of total charge enclosed by the Surface.

$$\Sigma Q = Q_1 + Q_2 + Q_3 - - - - Q_n$$

② Electric flux of the Surface depends upon charge enclosed by the surface.

③ Charge ⓟnt outside the surface doesn't affect the electric flux of Surface.

④ For ⊕ve charge electric flux will be outward and for ⊖ve charge electric flux will be inward.

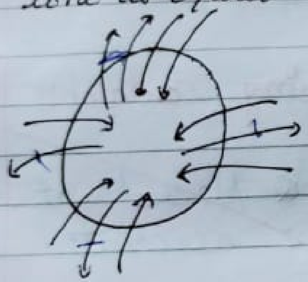
5. Outward flux will be treated as positive flux and inward flux will be treated as ⊖ve flux.
6. Total flux of surface is a sum of outward and inward flux.

$$\phi_{total} = \phi_{outward} + \phi_{inward}$$

$$\phi_{outward} = \oplus ve$$

$$\phi_{inward} = \ominus ve.$$

Que → 1 find total flux of given surface. if 1 flux line is equal to 2×10^5 volt-m.



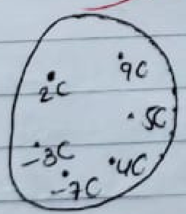
$\phi_{total} = 5 - 7$

$\phi_{total} = -2$ lines.

$\phi = -2\times 2\times10^5$ V-m

$\phi = -4\times10^5$ V-m

Que → 2

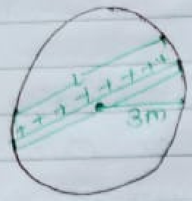


$$\phi = \frac{\Sigma Q}{\varepsilon_0} = \frac{+10}{8.85\times10^{-12}}$$

$$\phi = \frac{1}{8.85\times10^{-13}} \text{ V-m}$$

Que → 3 As shown in figure A charge rod of charge density of 3 c/m is passing through the centre of a sphere if radius is 3m then find out flux associated with charge.

$\lambda = \frac{Q}{L} = C/m$



$\lambda = 3\ C/m$

$L = 2r$

$\phi = \frac{\varepsilon Q}{\varepsilon_0}$

$\varepsilon Q = \lambda \cdot L$

$\varepsilon Q = \lambda \times 2r$

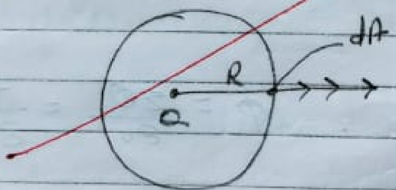
$\phi = \frac{\lambda \times 2r}{\varepsilon_0}$

$\phi = \frac{3 \times 2 \times 3}{8.85 \times 10^{-12}}$

$\phi = \frac{18 \times 10^{12}}{8.85}$

Derivation of Gauss's law using Coulomb's law

As shown in figure charge Q placed at center of sphere have electric flux associated with this sphere.

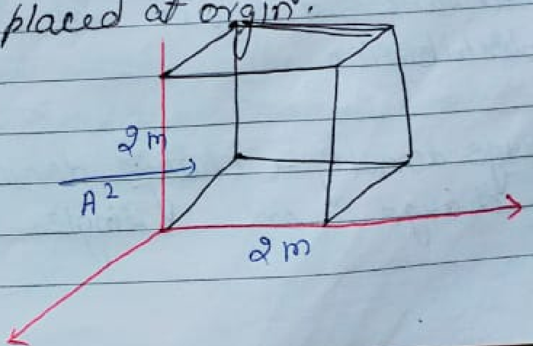


we know that $\phi = EA\cos\theta$
and in given figure electric field is variable at each and every point hence we will use a smaller particle dA.

So, $d\phi = E\,dA\cos\theta$

$$d\phi = EdA\cos\theta$$
$$\theta = 0^\circ$$
$$d\phi = EdA\cos 0^\circ$$
$$d\phi = EdA$$
$$\phi = \oint d\theta$$
$$\phi = \oint E \times dA$$
$$\phi = \oint \frac{KQ}{R^2}\, dA$$
$$\phi = \frac{KQ}{R^2} \oint dA$$
$$K = \frac{1}{4\pi\varepsilon_0}$$
$$\phi = \frac{1}{4\pi\varepsilon_0} \frac{Q}{R^2} \oint dA$$
$$\phi = \frac{1}{4\pi\varepsilon_0} \frac{Q}{R^2} [A_s] \quad [\text{Area of sphere} = 4\pi R^2]$$
$$\phi = \frac{Q}{\cancel{4\pi}\varepsilon_0 \cancel{R^2}} \times \cancel{4\pi R^2}$$
$$\phi = \frac{Q}{\varepsilon_0}$$

doubt
Que → ① If E-field is given by Exî & In this electric a cube of side 2m is placed then find out total flux associated with cube given that $E = 2 \times 10^5$ N/C & cube placed in x-direction starting face of cube is in Xy direction placed at orgin.



→ $\vec{E} = Ex\hat{i}$
$\vec{E} = 2 \times 10^5 x\hat{i}$

$$\phi = \phi_{s1} + \phi_{s2} + \phi_{s3} + \phi_{s4} + \phi_{s5} + \phi_{s6}$$

$$\phi = E_1 A_1 \cos\theta_1 + E_2 A_2 \cos\theta_2 + E_3 A_3 \cos\theta_3 + E_4 A_4 \cos\theta_4 + E_5 A_5 \cos\theta_5 + E_6 A_6 \cos\theta_6 .$$

$E_1 = Ex\hat{i}$ | $E_2 = Ex\hat{i}$

$E_1 = 2\times10^5 \times 0\ \hat{i}$ | $E_2 = 2\times10^5 \times 2$

$E_1 = 0$ | $E_2 = 4\times10^5$

$$A = \text{Side}^2$$

$$A = (2)^2 = 4$$

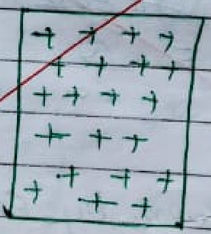
$\phi_3 = 90^\circ / \phi_4 = 90^\circ / \phi_5 = 90^\circ / \phi_6 = 90^\circ$

so, $\phi = E_2 A_2 \cos\theta_2$

$\phi = 4\times10^5 \times 4$

$\phi = 16\times10^5\ V\text{-}m$

Continuous or regular charge distribution and charge density.

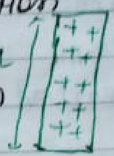


If an object charged by a large charge in such a way that object is charged at atomic level then this type of charge ~~con~~ or charge distribution is known as continous charge distribution.

Charge density → charge density is known as charge on single unit of object.

$$\text{Density} = \frac{\text{charge}}{\text{Unit}}$$

→ Different type of charge distribution

① linear or line charge distribution  $L >>> b / L >>> h$

If a linear object is charged by a large charge and this charge is distributed at atomic level of the linear object than this type of charge distribution is known as linear charge distribution

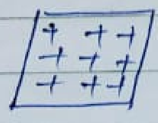
② Linear charge density - charge of unit length of a linear object is called linear charge density.

Represented by 'λ'.

$$\lambda = \frac{Q}{L}$$

→ unit = columb
→ Dimention = $[M^0 L^{-1} T^1 A^1]$

③ Surface charge distribution - If a 2 object is charged till it get charge at its atomic level then this type of charge is called surface charge distribution.

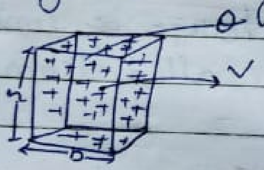


④ Surface charge density - Value of charge present in unit area of two object is called as Surface Charge density.

represented by 'σ'.

→ unit = C/m^2
→ Dimen = $[M^0 L^{-2} T^1 A^1]$

⑤ Volumatric charge distribution - If 3d object is charged till it get charge at its atomic level than this type of charge is called V.C.D.



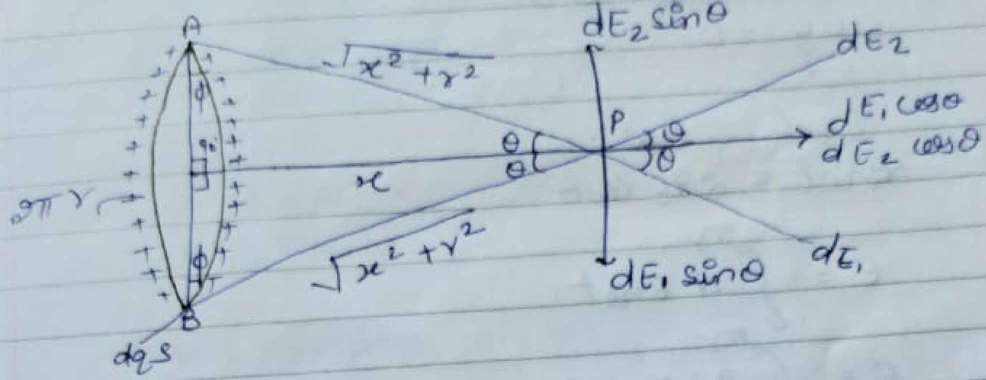
⑥ Volumetric charge Density - charge of unit Volume of an object is known as Volumetric charge density.

→ Represented by 'ρ'

$$\rho = \frac{\theta}{V}$$

→ Unit = C/m^3
→ Dimen = $[M^0 L^{-3} T^1 A^1]$

Electric field intensity due to ring



ΔABP is an isoscelles

$$AP = BP$$

$$\lambda = \frac{Q}{l}$$

$$E_{net} = \sqrt{E_x^2 + E_y^2}$$

$$E_x = dE_1\cos\theta + dE_2\cos\theta$$
$$E_y = dE_2\sin\theta - dE_1\sin\theta$$

$$dE_1 = \frac{k\,dq}{(\sqrt{r^2+x^2})^2}$$

$$dq = \lambda \times dl$$
$$dE_1 = \frac{k \times \lambda dl}{(\sqrt{x^2+r^2})^2} \quad \Big| \quad dE_2 = \frac{k\lambda dl}{(\sqrt{r^2+x^2})^2}$$

$$(dE_1) = |dE_2|$$

$$E_x = 2dE\cos\theta$$
$$E_y = 0$$

$$E_{net} = E_x$$
$$dE_{net} = 2dE\cos\theta$$

$$E = \int_0^{\pi r} dE_{net}$$

$$E = \int_0^{\pi r} 2dE\cos\theta$$

$$E = \int_0^{\pi r} \frac{2k\lambda dl}{(\sqrt{r^2+x^2})^2}\cos\theta$$

$$\cos\theta = \frac{B}{H} = \frac{x}{\sqrt{r^2+x^2}}$$

$$E = \int_0^{\pi r} \frac{2k\lambda dl}{(\sqrt{r^2+x^2})^2} \times \frac{x}{(\sqrt{x^2+r^2})}$$

$$E = \frac{2k\lambda x}{(r^2+x^2)^{3/2}} \int_0^{\pi r} dl.$$

$$E = \frac{2k\lambda x}{(r^2+x^2)^{3/2}} [l]_0^{\pi r}$$

$$E = \frac{2k\lambda x}{(r^2+x^2)^{3/2}} [\pi r - 0]$$

$$\lambda = \frac{Q}{2\pi r}$$

$$E = \frac{2kQ}{2\pi r} \cdot \frac{\pi r \cdot x}{(r^2+x^2)^{3/2}}$$

$$E = \frac{kQx}{(r^2+x^2)^{3/2}}$$

$$x >>>> r$$

$$x^2 >>>> r^2$$

$$r^2 + x^2 \approx x^2$$

$$E = \frac{kQx}{(r^2)^{3/2}}$$

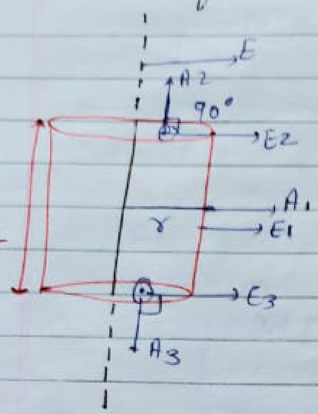
$$E = \frac{kQx}{r^3}$$

$$\boxed{E = \frac{kQ}{r^2}}$$

Charg = gasian surface
• Charg = spherical
line charge = cylindrical
Spherical charg = spherical.

→ Application of Gauss' law.

① # Electric field due to line charge. (∞ charged wire)



Gauss law →

$$\phi = \frac{\varepsilon Q}{\varepsilon_0}$$

$$\phi = \frac{\varepsilon Q}{\varepsilon_0}$$

charge density = λ

$$\lambda = \frac{Q}{l}$$

$$Q = \lambda l$$

$$\varepsilon Q = \lambda \times L$$

$$\phi_2 = \frac{\lambda L}{\varepsilon_0} \quad \text{— ①}$$

$$\phi = E ds \cos\theta$$

$$\phi = \phi_1 + \phi_2 + \phi_3$$

$$\phi_1 = E_1 A_1 \cos\theta_1$$

$$\phi_1 = E_1 A_1 \cos 0°$$

$$\phi_1 = E_1 A_1$$

$$\phi_2 = 0 \; (\because \theta = 90°)$$

$$\phi_3 = 0 \; (\text{'' '' ''})$$

$$\phi = E_1 A_1 + 0 + 0$$

$$A_1 = 2\pi r l$$

$$\phi = E \times 2\pi r l \quad \text{— ②}$$

by eq ① = eq ②

$$\frac{\lambda \times \cancel{L}}{\varepsilon_0} = E \times 2\pi r \cancel{l}$$

$$\frac{2 \times \lambda}{2 \times 2\pi r \varepsilon_0} = E$$

$\frac{2\lambda}{4\pi\epsilon_0 r} = E$

$$\boxed{E = \frac{2k\lambda}{r}}$$

Conductor → Substance which allows flow of charge is known as conductor.

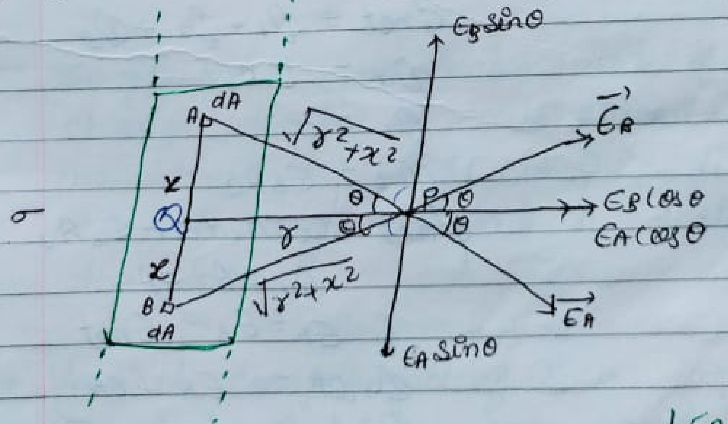
we can say that a substance which allows flow of current is known as conductor.

Note → Charge on a conductor flows over the surface or present at the surface.

Insulator → Substance which doesn't allow flow of charge is known as insulator.

Note → charge in an insulator ⊕nt at each and every point.

② # EFI due to an Insulating plate of ∞ length.



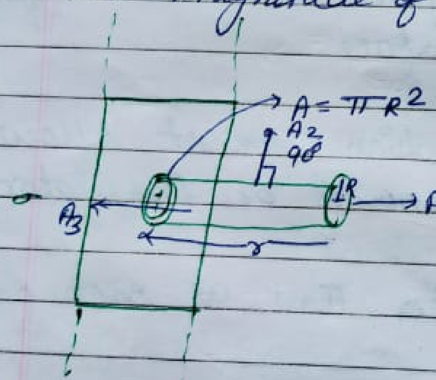
$|E_A| = |E_B|$

→ As shown in figure an insulating plate of ∞ length which charge density (σ) is placed at a distance of r from the plane at point P.
we have to find EFI at point P.

(3)

- For EFI at point P we assume to smaller area equidistance from point Q
due to these smaller area EFI will be ⊥ to the plate (as shown in fig)

→ For magnitude of EFI we apply Gauss' law.



→ By the def^n of Gauss' law

$$\phi_e = \frac{\Sigma Q}{\varepsilon_0}$$

$$\Sigma Q = \sigma \times A$$

$$\phi = \frac{\sigma \times A}{\varepsilon_0} \quad \text{—— (1)}$$

→ By the def^n of flux

$$\phi = EA\cos\theta$$

$$\phi_{net} = \phi_1 + \phi_2 + \phi_3$$

$$\phi_1 = E_1 A_1$$

$$\phi_2 = 0$$

$$\phi_3 = E_3 A_3$$

$$\phi_{net} = E_1 A_1 + 0 + E_3 A_3$$

$$A_1 = A_3 = A$$

$$E_1 = E_3 = E_{net}$$

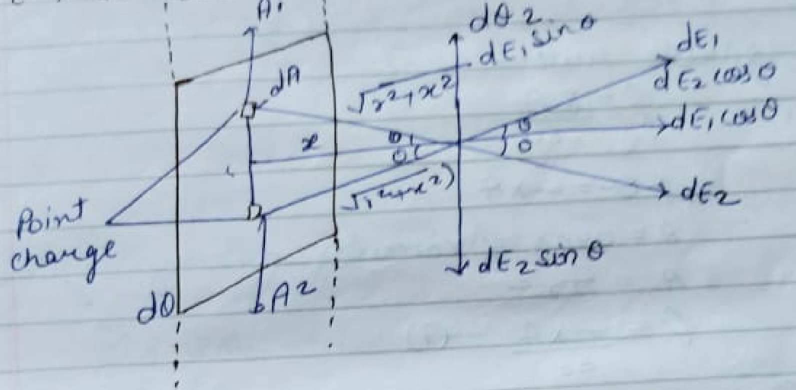
$$\phi_{net} = EA + EA$$

$$\phi_{net} = 2EA \quad \text{—— (2)}$$

$eq \to ① \& ②$

$E = \frac{\sigma}{2\varepsilon_0}$

③ E.F.I due to a conductor plate of ∞ length



$E_y = dE_1 \sin\theta - dE_2 \sin\theta$

$E_x = dE_1 \cos\theta + dE_2 \cos\theta$

$\frac{d\theta_1}{dE_1} = \frac{d\theta_2}{dE_2}$

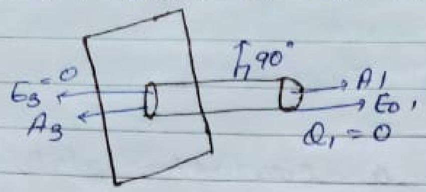
$dE_1 = dE_2$

$dy = 0$

$E_x = 2dE\cos\theta$

→ In figure, a conductor plate of ∞ length which charge density of σ is placed due to this plate we have to find electric field intensity at point O, which is at a distance of r from the plate.

→ Vertical components of E.F due to smaller particals A_1 & A_2 are canel out while horizontal components is added with each other. hence, net EF will be ⊥ to the plate.



By gauss's rule →

$$\phi = \frac{E\theta}{\varepsilon_0}$$

$$\Sigma\theta = \sigma \times A$$

A = area of circular area of cylinder

$$A = \pi r^2$$

$$\phi = \frac{\sigma \times A}{\varepsilon_0} \quad \text{—①}$$

By defⁿ of electric fluse

$$\phi = E\,d\cos\theta$$

$$\phi = \phi_1 + \phi_2 + \phi_3$$

$$\phi_1 = E_1 A_1 \cos\theta_1$$

$$\phi_1 = E_1 A_1$$

$$\phi_2 = E_2 A_2 \cos\theta_2 \quad (\theta = 90)$$

$$\phi_2 = 0$$

$$\phi_3 = E_3 A_3 \cos\theta_3 \quad (E_3 = 0)$$

$$\phi_3 = 0$$

$$\phi_{net} = E_1 A_1 + 0 + 0$$

$$\phi_{net} = EA \quad \text{—②}$$

by eq ① and ②

$$EA = \frac{\sigma \times A}{\varepsilon_0}$$

$$\boxed{E = \frac{\sigma}{\varepsilon_0}}$$

by eq-① and eq-②

$$\frac{\sigma \times 4\pi R^2}{\varepsilon_0} = E \times 4\pi r^2$$

$$\boxed{E = \frac{\sigma R^2}{\varepsilon_0 r^2}}$$

$$\boxed{E \propto \frac{1}{r^2}}$$

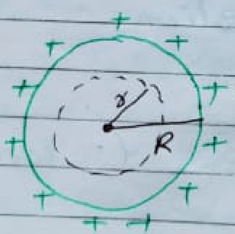
Ⓑ $(r = R)$ on the surface.

$\because$ $R = r$

$\therefore$ $E = \frac{\sigma \cancel{R^2}}{\varepsilon_0 \cancel{R^2}}$

$$\boxed{E = \frac{\sigma}{\varepsilon_0}}$$

Ⓒ $r < R$ (inside the surface)



→ flux using Gauss's law

$$\phi = \frac{\Sigma Q}{\varepsilon_0}$$

$\Sigma Q = 0$ (charge on Gaussian surface is 0)

$$\phi = 0 \quad \text{—} (1)$$

→ flux using flux's defination.

$$d\phi = E\,dA\cos\theta$$

$$d\phi = E\,dA$$

$$\phi = \oint_S E\,dA$$

$$\phi = E\oint_S dA$$

$$\phi = E[A]_S$$

$$\phi = E \times 4\pi r^2 \quad \text{—} (2)$$

by eq (1) and eq (2)

$$0 = E \times 4\pi r^2$$

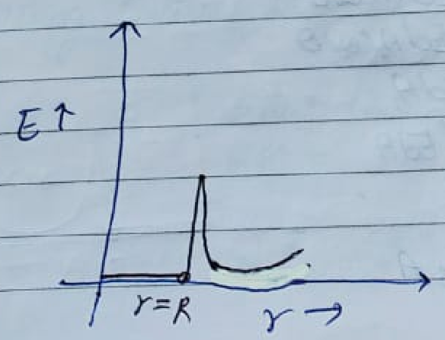
$$\boxed{E = 0}$$

→ Graph b/w E and r

$$E_{out} \propto \frac{1}{r^2} = \frac{\sigma R^2}{\varepsilon_0 r^2}$$

$$E_{surface} = \text{Constant} = \frac{\sigma}{\varepsilon_0}$$

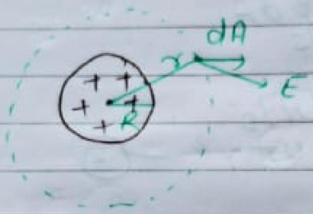
$$E_{inside} = 0 \quad (r < R)$$



(5) Electric field due to Solid Sphere :-

a) Conducting Solid Sphere :- charge provided to a Conducting solid sphere will redistributed on the surface because Conductors have property to redistribute charge on the surface therefore Solid conducting sphere will act like a hollow sphere.

b) Insulating Solid sphere : -



Case-I If $r > R$ (outside the surface)

using gauss law $\rightarrow$ $\phi = \frac{\Sigma Q}{\varepsilon_0}$ $\left(\because \rho = \frac{Q}{V}\right)$

$$\varphi = \rho \times V$$

$$\Sigma Q = \rho \times \frac{4}{3}\pi R^3$$

$$\phi = \frac{\rho \times \frac{4}{3}\pi R^3}{\varepsilon_0} \quad \text{— (1)}$$

using flux defination

$$d\varphi = E\,dA\cos\theta$$

$$d\varphi = E\,dA\cos\theta$$

$$d\varphi = E\,dA$$

$$\varphi = \oint_s E\,dA$$

$$\varphi = E \oint_s A$$

$$\phi = E[A]s$$

$$\phi = E \times 4\pi r^2 \quad \text{—(2)}$$

by eq-(1) & (2)

$$\frac{\rho \times \frac{4}{3}\pi R^3}{\varepsilon_0} = E \times 4\pi r^2$$

$$\frac{\rho \times \frac{R^3}{3}}{\varepsilon_0} = E \times r^2$$

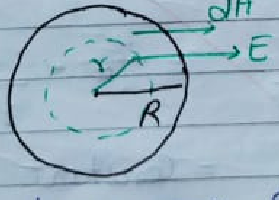
$$\boxed{E = \frac{\rho \times R^3}{3\varepsilon_0 \times r^2}}$$

Case-II $r = R$ (on the surface)

$$E = \frac{\rho R^3}{3\varepsilon_0 R^2}$$

$$\boxed{E = \frac{\rho R}{3\varepsilon_0}}$$

Case-III $r < R$ (inside the surface)



Using Gauss law $\rightarrow \phi = \frac{\Sigma q}{\varepsilon_0}$

$$\Sigma q = \rho \times V$$

$$\Sigma \phi = \rho \times \frac{4}{3}\pi r^3$$

$$\phi = \frac{\rho \times \frac{4}{3}\pi r^3}{\varepsilon_0} \quad \text{—(1)}$$

using Gauss law :- $\phi = \frac{\varepsilon Q}{\varepsilon_0}$

$\varepsilon Q = \rho \times V$

using flux defination ($\theta = 0$)

$d\phi = E\,dA\cos\theta$

$d\phi = E\,dA\cos\theta$

$d\phi = E\,dA$

$\phi = \oint_s E\,dA$

$\phi = E[A]_s$

$\phi = E \times 4\pi r^2$ —— (2)

by eq (1) & (2)

$$\frac{\rho \times \frac{4}{3}\pi r^3}{\varepsilon_0} = E \times 4\pi r^2$$

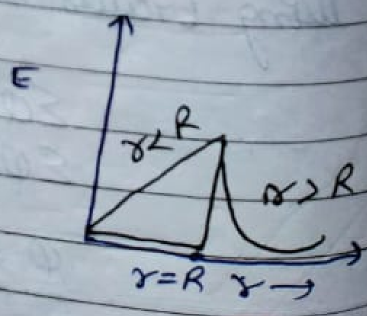
$$\boxed{\frac{\rho \times r}{3\varepsilon_0} = E}$$

Graph b/w E and r.

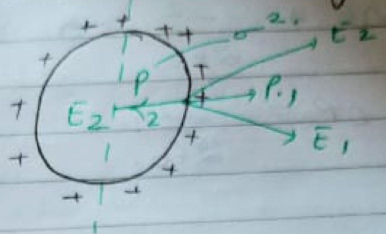
$E_{out} = E = \frac{\rho R^3}{3\varepsilon_0 r^2} = E \propto \frac{1}{r^2}$

$E_{surface} = \frac{\rho R}{3\varepsilon_0}$ = Constant

$E_{inside} = \frac{\rho r}{3\varepsilon_0} = E \propto r$



→ Pressure difference in a charged conductor sphere



$E_{P1} = E_1 + E_2$ — (1)

$E_{P2} = E_1 - E_2$ — (2)

$E_{P2} = 0$

$E_1 - E_2 = 0$

$E_1 = E_2$

by putting values of E_1 & E_2 in eq — (1)

$E_{P1} = E + E$

$E_{P1} = 2E$

$E_{P1} = \frac{\sigma}{\varepsilon_0}$ (on the surface of hollow sphere)

So, $E = \frac{E_{P1}}{2}$

$$E = \frac{\sigma}{2\varepsilon_0} \quad -(2)$$

force $= E \times q$ $\quad [\because E = \frac{f}{q}]$

$$f = \frac{\sigma}{2\varepsilon_0} \times q$$

for Pressure $= \frac{f}{A}$

$$P = \frac{\sigma}{2\varepsilon_0} \times \frac{q}{A}$$

$$P = \frac{\sigma \times \sigma}{2\varepsilon_0} \qquad \left(\sigma = \frac{q}{A}\right)$$

$$\boxed{P = \frac{\sigma^2}{2\varepsilon_0}}$$

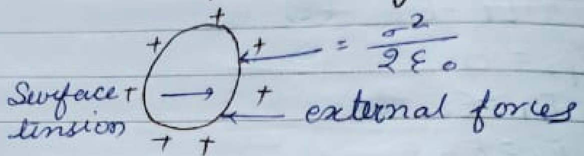
→ Pressure inside $= 0$

$$\Delta P = P_{outside} - P_{inside}$$

$$\Delta P = \frac{\sigma^2}{2\varepsilon_0} - 0$$

$$\Delta P = \frac{\sigma^2}{2\varepsilon_0}.$$

→ Equili Condition of a charged soap bubble



Equili condi →

So $P_{net} = 0$

$$\frac{\sigma^2}{2\varepsilon_0} + P_{ext} = \text{Surface tension}$$

due to stability $P_{ext} = 0$

$$\frac{\sigma^2}{2\varepsilon_0} = \text{surface tension.}$$

* Surface tension due to bubble $= \frac{4T}{R}$.

$$\frac{\sigma^2}{2\varepsilon_0} = \frac{4T}{R}$$

Surface tension.

$$\frac{\sigma^2}{2\varepsilon_0} = \frac{4T}{R}$$

* $T = \frac{\sigma^2 R}{2\varepsilon_0 \times 4}$

$$T = \frac{\sigma^2 R}{8\varepsilon_0}$$

$$\sigma^2 = \frac{4T \times 2\varepsilon_0}{R}$$

$$\sigma = \sqrt{\frac{8\varepsilon_0 T}{R}}$$

* $R = \frac{\sigma^2}{2\varepsilon_0 4T}$

$$R = \frac{\sigma^2}{8\varepsilon_0 T}$$

01/08